

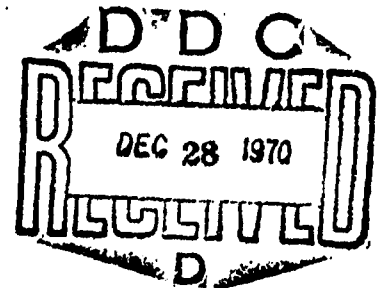
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The Open-Circuit Sensitivity Of Axially Polarized Ferroelectric Ceramic Cylinders

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22 September 1970



NAVAL UNDERWATER SYSTEMS CENTER
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ABSTRACT

The open-circuit sensitivities of axially polarized ferroelectric ceramic cylinders are derived by treating the ceramic as an anisotropic material. Although the results for cylinders that are pressure-released at the inside lateral surface were identical to those given by Langevin, the analysis showed that the anisotropic properties of currently used ceramics did not influence the sensitivities of these types of cylindrical hydrophone elements.

Axially poled cylinders that were completely pressure-released with respect to the radial coordinate were found to exhibit sensitivities larger than those of similar cylinders that were pressure-released on only one major surface, the inside lateral surface. Included in this report are graphs of the magnitude of the stresses induced in the cylinders by the ambient hydrostatic pressures.

ADMINISTRATIVE INFORMATION

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THE OPEN-CIRCUIT SENSITIVITY OF AXIALLY POLARIZED FERROELECTRIC CERAMIC CYLINDERS

INTRODUCTION

The open-circuit sensitivities of axially polarized cylinders with capped, exposed, and shielded ends were investigated in order to compare different cylindrical hydrophone designs. Although the analysis is approached from the viewpoint of treating an anisotropic material, which is the necessary approach in treating polarized ferroelectric materials, the results are identical to those advocated by Langevin.¹ The anisotropic properties of the material did not introduce a paradox similar to the one experienced for a radially polarized ceramic cylinder.²

THEORETICAL CONSIDERATIONS

Figure 1 depicts a right circular cylinder of polarized ferroelectric ceramic. By convention, the principal directions of stress in the material are designated by the axes 1, 2, and 3 and are chosen to coincide with the set of cylindrical coordinates r , θ , and z , respectively. Axis 3 is parallel to the polarization vector (axial) in the ceramic.

At the outset, circular symmetry will be assumed, such that the tangential particle displacement (u_θ) and any differentiations with respect to the angular coordinate (θ) can be neglected. One way of expressing the linear equations of state of the material is as follows:

$$S_1 = s_{11}^D T_1 + s_{12}^D T_2 + s_{13}^D T_3 + g_{31} D_3, \quad (1)$$

$$S_2 = s_{12}^D T_1 + s_{11}^D T_2 + s_{13}^D T_3 + g_{31} D_3, \quad (2)$$

$$S_3 = s_{13}^D T_1 + s_{13}^D T_2 + s_{33}^D T_3 + g_{33} D_3, \quad (3)$$

$$S_5 = s_{44}^D T_5, \quad (4)$$

and

$$E_3 = -g_{31} T_1 - g_{31} T_2 - g_{33} T_3 + \beta_{33}^T D_3, \quad (5)$$

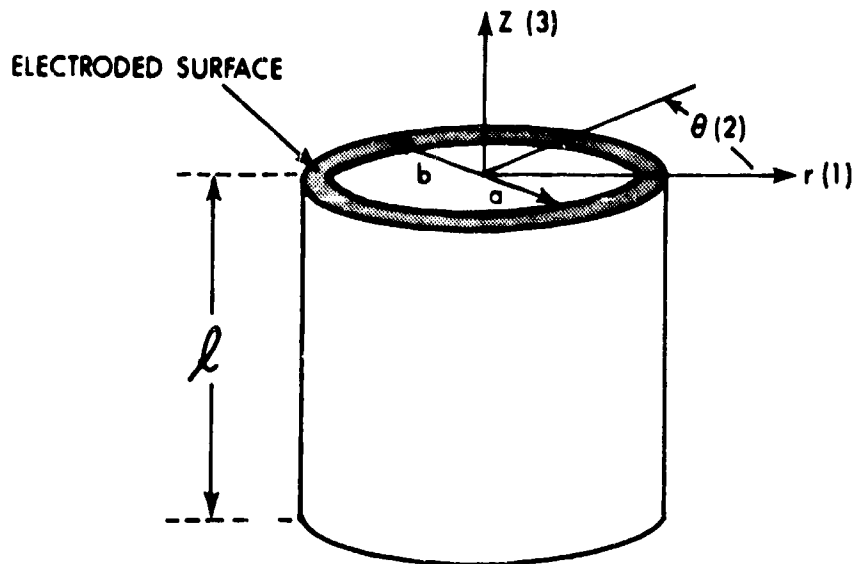


Fig. 1 - A Right Circular Cylinder of Axially Polarized Ferroelectric Ceramic

where s^D is the elastic compliance coefficient at constant electric displacement, g is the piezoelectric coefficient, and β_{33}^T is the incremental impermeability at constant stress (T). In cylindrical coordinates, the strains are expressed as

$$S_1 = \frac{\partial u_r}{\partial r}, S_2 = \frac{u_r}{r}, S_3 = \frac{\partial u_z}{\partial z}, \text{ and } S_5 = \frac{\partial u_r}{\partial z} + \frac{\partial u_z}{\partial r}, \quad (6)$$

where u_r and u_z are the radial and axial particle displacements, respectively.

Before proceeding further, let us note some of the constraints imposed upon the electric displacement (\vec{D}) and the electric field (\vec{E}). Maxwell's equations, neglecting the influence of the time rate of change of magnetic induction and the presence of local concentrations of free charge in the ceramic, require that the curl of the electric field and the divergence of the electric displacement be zero.

When cylindrical coordinates are used, these constraints may be expressed mathematically as

$$\vec{\nabla} \cdot \vec{D} = \frac{1}{r} \left[\frac{\partial}{\partial \theta} (r D_\theta) + \frac{\partial}{\partial z} (r D_z) \right] + \frac{1}{r} \left[\frac{\partial}{\partial z} (r D_r) - \frac{\partial}{\partial r} (r D_z) \right] = 0, \quad (7)$$

and

$$\vec{\nabla} \cdot \vec{D} = \frac{\partial D_1}{\partial r} + \frac{D_1}{r} + \frac{1}{r} \frac{\partial D_2}{\partial \theta} + \frac{\partial D_3}{\partial z} = 0, \quad (8)$$

where $\vec{1}_r$, $\vec{1}_\theta$, and $\vec{1}_z$ represent unit vectors in the r , θ , and z directions, respectively.

Since only the lateral surfaces perpendicular to the cylinder axis at $z = 0$ and l are electroded, such that $\mathcal{E}_1 = \mathcal{E}_2 = 0$, then Eq. (7) states that

$$\frac{\partial \mathcal{E}_3}{\partial \theta} = 0 \quad \text{and} \quad \frac{\partial \mathcal{E}_3}{\partial r} = 0. \quad (9)$$

Therefore, \mathcal{E}_3 must be a spatial constant with respect to the coordinate variables θ and r . When similar reasoning is employed, Eq. (8) requires that D_3 be a spatial constant with respect to the z coordinate. However, not one of the above equations gives any information concerning the possible dependence of D_3 on the radial coordinate (r). Some insight into this possible dependence can be gained by rearranging Eq. (5) to express D_3 in terms of \mathcal{E}_3 and the principal stresses (T_1 , T_2 , and T_3), such that

$$D_3 = \frac{\mathcal{E}_3}{\beta_{33}^T} + \frac{\beta_{31}}{\beta_{33}^T} (T_1 + T_2) + \frac{\beta_{33}}{\beta_{33}^T} T_3. \quad (10)$$

Differentiating Eq. (10) with respect to the radial coordinate yields

$$D_3' = \frac{\beta_{31}}{\beta_{33}^T} (T_1' + T_2') + \frac{\beta_{33}}{\beta_{33}^T} T_3', \quad (11)$$

where the prime means $\partial/\partial r$ and \mathcal{E}_3' have been set equal to zero, as required by Eq. (9).

In order to determine whether or not D_1 depends on the radial coordinate, we must divert our attention momentarily to the investigation of the principal stresses and their spatial distributions. If Eq. (2), in conjunction with Eq. (6), is used to define the radial displacement (u_r) and is differentiated once with respect to the radial coordinate, then

$$\frac{\partial u_r}{\partial r} = s_{12}^D T_1 + s_{11}^D T_2 + s_{13}^D T_3 + \beta_{11} D_1 + r \left[s_{12}^D T_1' + s_{11}^D T_2' + s_{13}^D T_3' + \beta_{11} D_1' \right]. \quad (12)$$

Equating the above result with Eq. (1), remembering that $S_1 = \partial u_r / \partial r$, from Eq. (6), yields

$$(s_{12}^D - s_{11}^D)T_1 + (s_{11}^D - s_{12}^D)T_2 + rs_{12}^D T_1' + rs_{11}^D T_2' + rs_{13}^D T_3' + r s_{31}^D D_3' = 0 \quad (13)$$

The equations of static equilibrium for a cylinder with circular symmetry and zero body forces may be written, assuming the shear stress $T_3 = 0$, as

$$\frac{\partial T_1}{\partial z} = 0, \quad \frac{1}{r} \frac{\partial T_2}{\partial \theta} = 0, \quad \text{and} \quad \frac{\partial T_1}{\partial r} + \frac{1}{r}(T_1 - T_2) = 0 \quad (14)$$

Because of the symmetry involved, the second equation of the above set is automatically satisfied. The first equation requires that the axial stress (T_1) be constant as far as the z coordinate is concerned.

In all the cases that shall be treated herein, the boundary conditions will require that the axial load be constant in terms of the radial coordinate at $z = 0$ and $z = l$. Therefore, $\partial T_1 / \partial r = 0$ will be a necessary condition throughout the body of the ceramic for all cases treated.

If the last equation in the above set is used to define the tangential stress in terms of the radial stress, such that

$$T_2 = rT_1' + T_1 \quad (15)$$

or

$$T_2' = rT_1'' + 2T_1' \quad (16)$$

then Eq. (13) may be rewritten as

$$T_1'' + \frac{1}{r} T_1' + \frac{s_{11}^D}{rs_{11}^D} D_3' = 0 \quad (17)$$

Now, if Eq. (11) is incorporated into Eq. (17), noting from previous considerations that $T_1 = 0$, then

$$T_1'' + \frac{1}{r} T_1' + \frac{s_{11}^D}{rs_{11}^D} \left[\frac{s_{11}}{s_{11}^D} (rT_1'' + T_1') \right] = 0 \quad (18)$$

or

$$T_1'' + \frac{3}{r} T_1' = 0. \quad (19)$$

The general solution of Eq. (19) is

$$T_1 = a_1 + a_2 r^{-2}, \quad (20)$$

such that the tangential stress distribution from Eq. (15) becomes

$$T_2 = a_1 - a_2 r^{-2}. \quad (21)$$

Since Eqs. (20) and (21) require that the radial and tangential stresses be solely a function of the radial coordinate, it is quite easy to prove that the shear stress in the plane perpendicular to the axial polarization vector is identically zero. The proof that the above condition exists is a necessity since the static equations of equilibrium that were used to establish the relationship between the radial and tangential stresses would be altered if the condition was not satisfied. More explicitly, for $S_r = 0$, Eq. (6) requires that $\partial u_r / \partial z = -\partial u_z / \partial r$. Inspection of Eq. (2), with the added requirements that T_1 and T_2 are functions of the radial coordinate only and that T_1 is to be considered constant for the cases treated herein, reveals that $\partial u_r / \partial z = 0$, if it is remembered that $\partial D_r / \partial z = 0$, as specified by Eq. (8) [$D_1 = D_2 = 0$].

When the previously stated assumptions are employed, Eq. (3) may be evaluated for the axial particle displacement as

$$u_z = \left[x_{11}^E (T_1 + T_2) + x_{11}^D T_1 + k_{11} D_1 \right] z + g(r). \quad (22)$$

In order to satisfy the criterion that the shear stress (T_r), and in turn the shear strain (S_r), is identically zero, then $\partial u_r / \partial r = 0$. This additional constraint can be satisfied for all values of the axial coordinate (z) if $g(r)$ equals a constant and $T_1 = -T_2$. (Previously it had been stipulated that $T_1 = 0$.) It is immediately obvious from Eqs. (20) and (21) that the imposed condition is ideally satisfied and that no paradox exists as was experienced for the case of radially polarized ferroelectric ceramic cylinders.² More explicitly, Eq. (22) may be rewritten, after differentiation with respect to the radial coordinate and with the aid of Eq. (11), as

$$\frac{\partial u_z}{\partial r} \left\{ T_1' \left[s_{13}^D + \frac{g_{33}g_{31}}{\beta_{33}^T} \right] + T_2' \left[s_{13}^D + \frac{g_{33}g_{31}}{\beta_{33}^T} \right] \right\} z=0 \quad (23)$$

or

$$T_1' + T_2' = 0. \quad (24)$$

Since it is obvious that Eq. (24) is ideally satisfied, then Eq. (11) states also that $D_3 = 0$ for the cases treated herein, or, more emphatically, that D_3 is also constant with respect to the radial coordinate. Therefore, the electric charge (q) may be derived as

$$q = \int_A D_3 dA = \int_a^{2\pi b} D_3 r dr d\theta = \pi D_3 (b^2 - a^2). \quad (25)$$

or

$$D_3 = \frac{q}{\pi(b^2 - a^2)}. \quad (26)$$

In all the cases treated herein, it will be assumed that the electrical terminals are open-circuited ($q = 0$), which implies that the electric displacement may be neglected for all practical purposes. This result could have been used initially to delete the electric displacement from the constitutive Eqs. (1) through (5), and the analysis could have been started from there by using this new set of equations. However, the approach that was used has shown that all the initial assumptions have been satisfied, and, therefore, the open-circuit sensitivities can be determined by using standard techniques.

The standard procedure for determining the open-circuit sensitivity of hydrophone elements is to define the sensitivity as the ratio of the magnitude of the open-circuit voltage to the magnitude of the free-field pressure that impinges upon the element. The sensitivities calculated herein are valid only for frequencies below and reasonably well removed from the lowest resonance frequency of the structure being considered.

At this point in the analysis it becomes convenient to consider specific cases, of which there are three of practical interest. The first case to be considered will be the most general since the other cases

are simply specializations of this one. Consider the case of an end-capped cylinder where the lateral surfaces perpendicular to the cylinder axis at $z = 0$ and l are closed by rigid seals in such a manner that radial motion is not impeded but that the cross section between $r = a$ and $r = b$ is subjected to an axial load of $-P_o / [1 - (a/b)^2]$, where P_o is the acoustic pressure expressed in dynes per square centimeter. From Eq. (14), since the first equation in the set requires that T_3 be constant in z ,

$$T_3 = \frac{-P_o}{\left[1 - \left(\frac{a}{b}\right)^2\right]} \quad (27)$$

throughout the ceramic body for this particular case.

Boundary conditions must now be imposed on the radial stress distribution. These will be established in the following manner: The inside lateral surface will be assumed to be completely shielded from the acoustic field such that $T_1 = 0$ at $r = a$. The outside lateral surface, however, is assumed to be exposed to a uniform acoustic pressure ($-P_o$) such that $T_1 = -P_o$ at $r = b$. When the above conditions are applied to the radial stress, Eq. (20), it is necessary that

$$a_2 = -a_1 a^2 \quad (28)$$

and

$$a_1 = \frac{-P_o}{\left[1 - \left(\frac{a}{b}\right)^2\right]} \quad (29)$$

Now the remaining principal stresses can be written in terms of the acoustic pressure as

$$T_1 = -P_o \frac{\left[1 - \left(\frac{a}{r}\right)^2\right]}{\left[1 - \left(\frac{a}{b}\right)^2\right]} \quad (30)$$

and

$$T_2 = -P_o \frac{\left[1 + \left(\frac{a}{r}\right)^2\right]}{\left[1 - \left(\frac{a}{b}\right)^2\right]} \quad (31)$$

The open-circuit voltage appearing between the electroded surfaces can be obtained from Eq. (5) through the following formula:

$$V_{oc} = \int_0^l \mathcal{E}_3 dz - \int_0^l \left[-g_{31} (T_1 + T_2) - g_{33} T_3 \right] dz \quad (32)$$

Substituting Eqs. (27), (30), and (31) into Eq. (32) gives the open-circuit voltage for an end-capped cylinder as

$$V_{oc} = \frac{P_o l}{\left[1 - \left(\frac{a}{b} \right)^2 \right]} (2g_{31} + g_{33}) \quad (33)$$

Therefore, the open-circuit sensitivity (M_o) for this case becomes

$$M_o = \frac{l (2g_{31} + g_{33})}{\left[1 - \left(\frac{a}{b} \right)^2 \right]} \quad (34)$$

The second case that will be considered is that of a cylinder where the lateral surfaces at $z = 0$ and l are exposed to the acoustic field, such that $T_3 = -P_o$ at these extreme points. This case is simply a special circumstance of the more general situation that was treated in the previous section of this report. In other words, the radial and tangential stress distributions, as given by Eqs. (30) and (31), are the same for this case, and the axial stress distribution becomes simply $T_3 = -P_o$. Therefore, by using the standard procedures, the open-circuit sensitivity for an exposed-end cylinder becomes

$$M_o = l \left\{ \frac{2g_{31}}{\left[1 - \left(\frac{a}{b} \right)^2 \right]} + g_{33} \right\} \quad (35)$$

The final case that will be considered is that of a cylinder where the lateral surfaces at $z = 0$ and l are completely shielded from the acoustic field, such that $T_3 = 0$. As a result, the open-circuit sensitivity for a shielded-end cylinder becomes

$$M_o = \frac{2l g_{31}}{\left[1 - \left(\frac{a}{b} \right)^2 \right]} \quad (36)$$

The expressions for the open-circuit sensitivities of axially polarized ferroelectric ceramic cylinders, Eqs. (34) through (36), are identical to those derived by Langevin. Although Langevin plotted

sensitivity curves for barium titanate ceramic only, the information contained in his formulas has been updated and can be readily found in other articles.³ It will be duplicated here for convenience. In addition, the information inherent in Eqs. (34) through (36) has been applied to a substantially new type of ceramic material, namely, lead metaniobate. See Figs. 2 through 5 for a graphic description of the above data. The values for lead metaniobate are based upon the manufacturer's literature.⁴ The preceding information has been presented solely for comparing different hydrophone designs. This particular aspect will not be treated in this presentation, but at least the necessary information will be available for future reference.

The values of sensitivity plotted in this report for the ceramic materials Ceramic B, PZT (hard lead zirconate titanate)-4, and PZT-5A are slightly higher than those presented by Berlincourt and Krueger.³ However, the computed values were based on the material properties published by the Clevite Corporation.³

As is readily seen from Figs. 2 through 5, the greatest sensitivities are achieved when the cylinder walls are very thin. However, this same region has the greatest variation in sensitivity because of the steep slopes that exist there and, therefore, would conceivably pose a problem as far as the uniformity of many specimens is concerned. In addition, very thin-walled specimens are susceptible to material parameter variations and failure due to fracture or depolarization because of the induced directional stresses set up throughout the material when it is subjected to ambient hydrostatic pressure. (See Appendix A.)

Good sensitivities can be achieved with solid rods that are polarized axially, especially for the cases of shielded-end cylinders of PZT-4 or PZT-5A and capped-end cylinders of lead metaniobate. As the thickness-to-outside-diameter ratio approaches 0.5, the slopes of the sensitivity curves are relatively flat; therefore, slight variations in the dimensions of solid rods should not cause drastic differences in the open-circuit sensitivity.

It should also be noted that the sensitivity of a capped-end cylinder of lead metaniobate is greater than the sensitivity of an exposed or shielded-end cylinder of the same material. But the opposite is true for the other materials when the thickness-to-outside-diameter ratio is less than 0.15. This phenomenon is due to the large value of the piezoelectric constant (g_{33}) that the lead metaniobate exhibits.

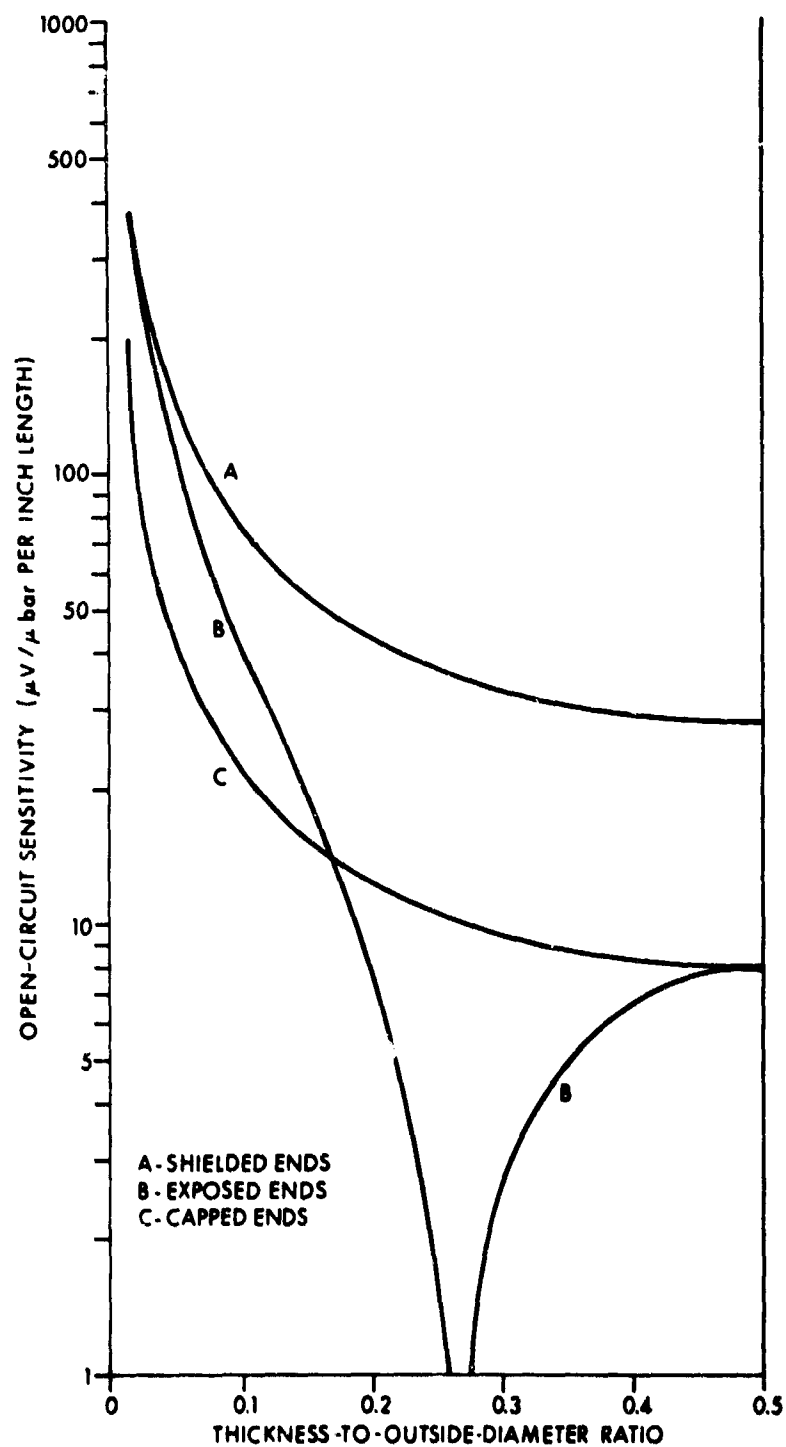


Fig. 2 - Open-Circuit Sensitivity versus Thickness-to-Outside-Diameter Ratio of an Axially Polarized Ceramic B Cylinder

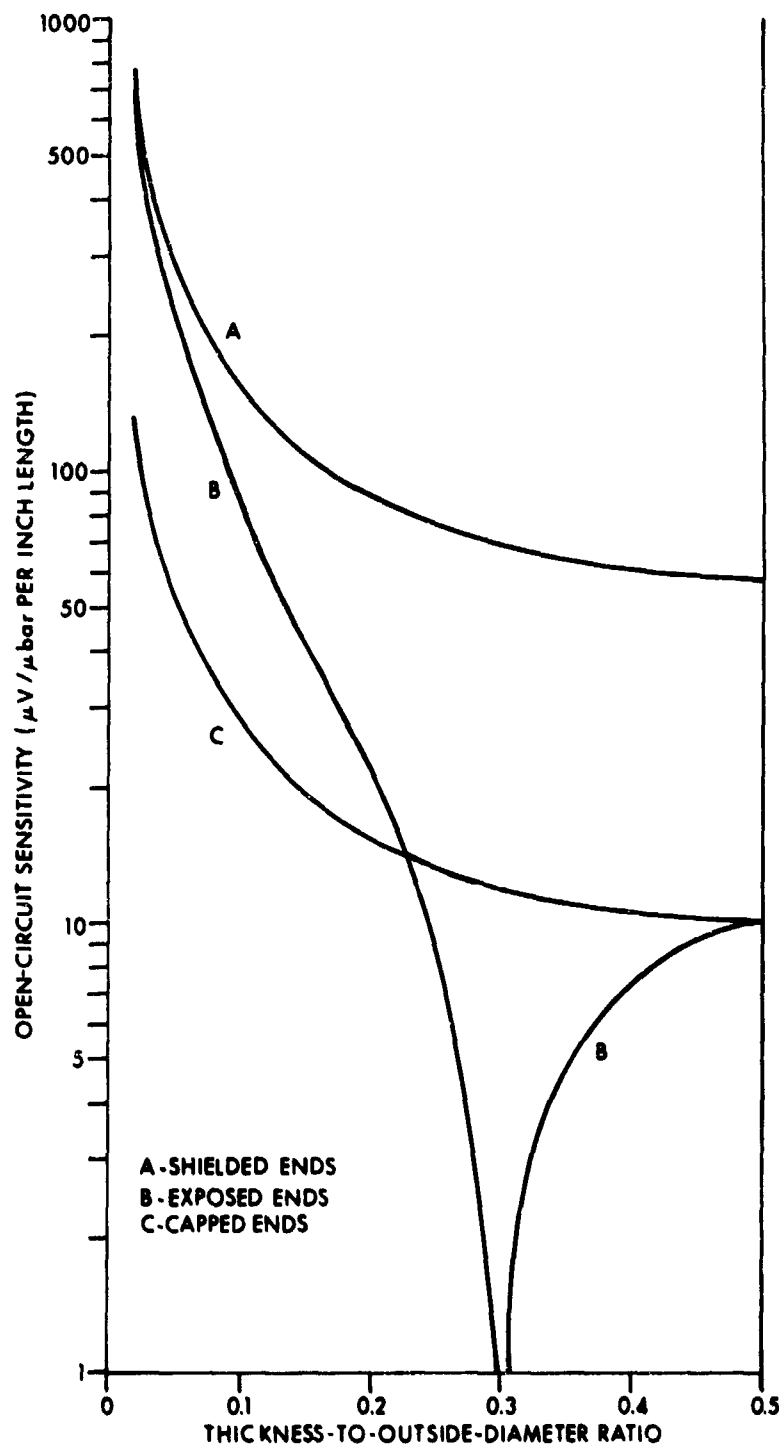


Fig. 3 - Open-Circuit Sensitivity versus Thickness-to-Outside-Diameter Ratio of an Axially Polarized PZT-4 Cylinder

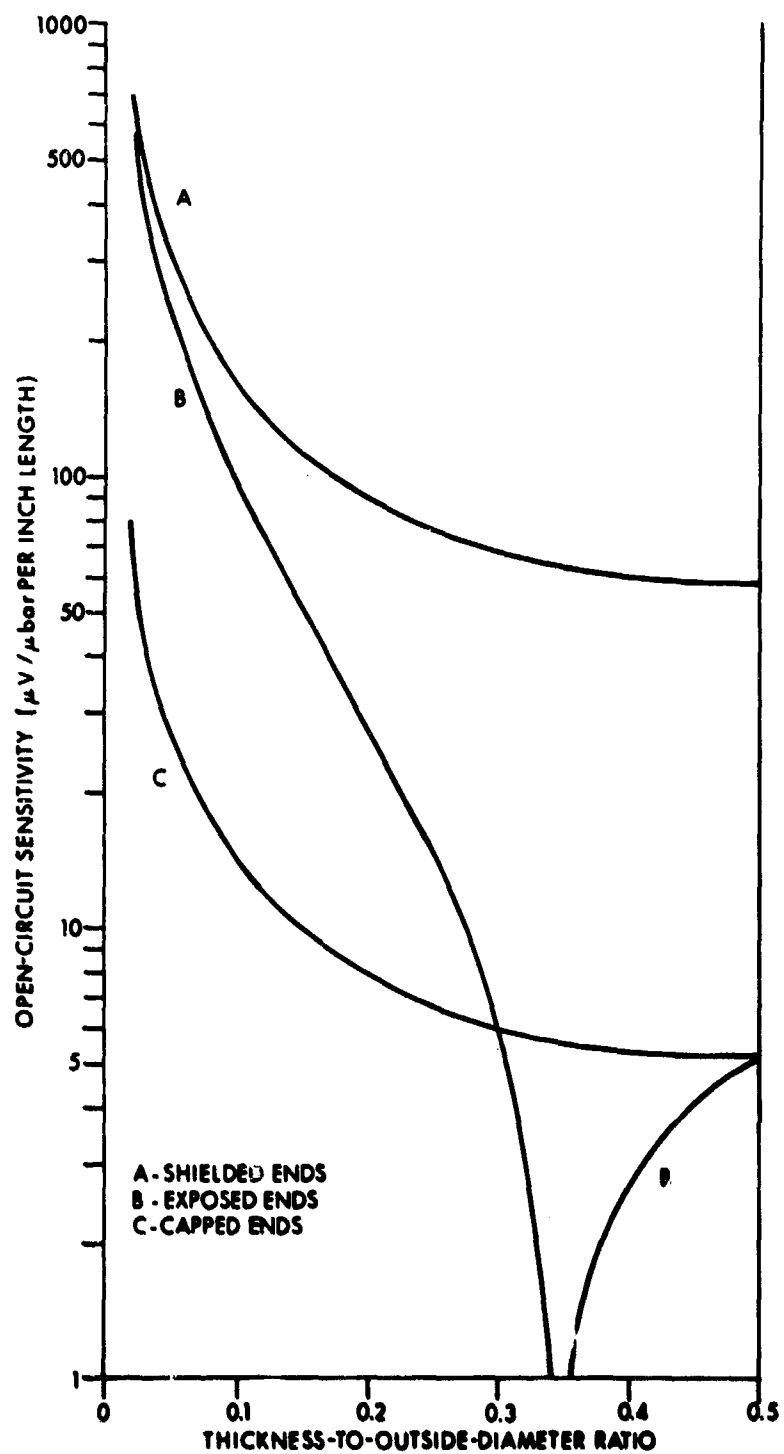


Fig. 4 - Open-Circuit Sensitivity versus Thickness-to-Outside-Diameter Ratio of an Axially Polarized PZT-5A Cylinder

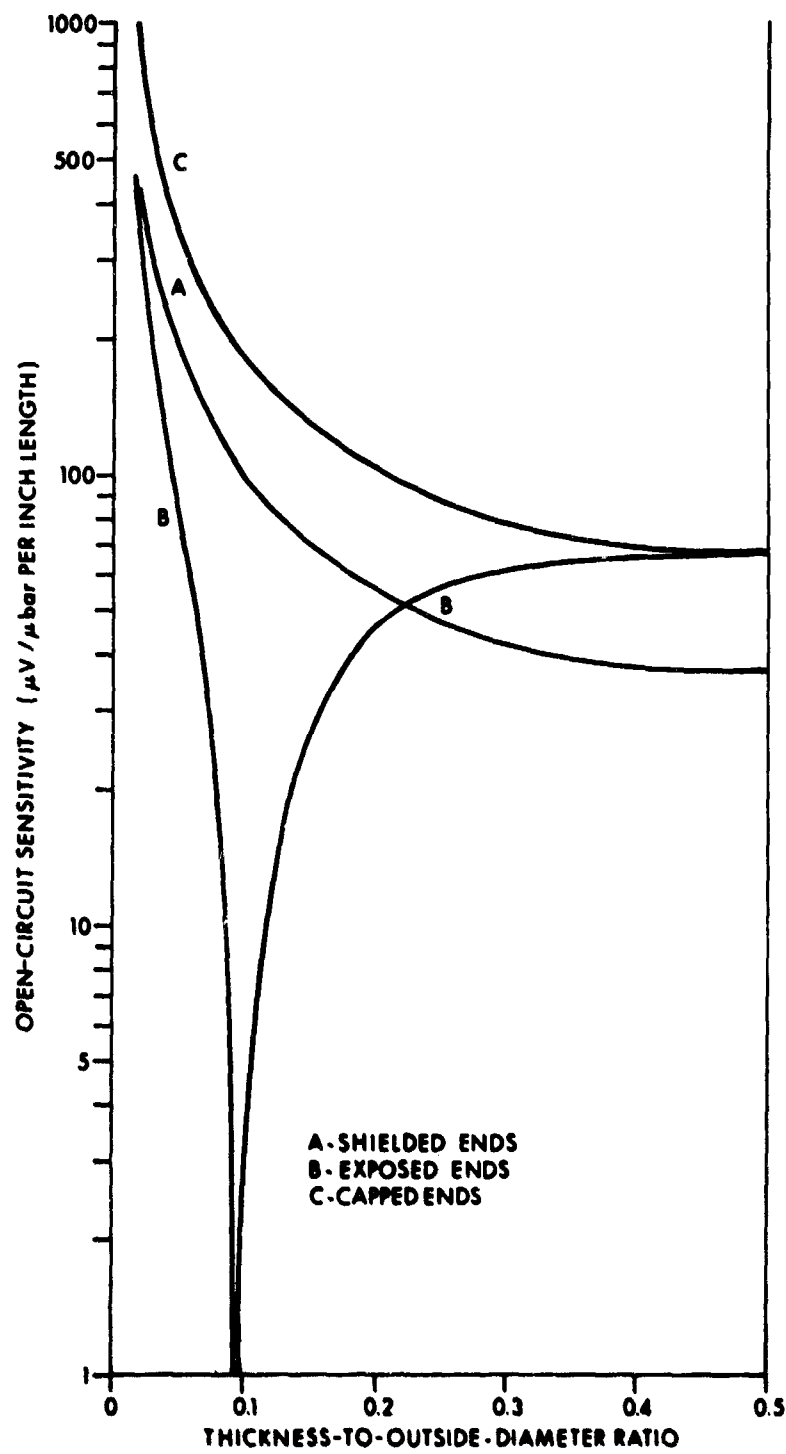


Fig. 5 - Open-Circuit Sensitivity versus Thickness-to-Outside-Diameter Ratio of an Axially Polarized Lead Metaniobate Cylinder

COMMENTS

The open-circuit sensitivities of three different configurations of axially polarized ferroelectric ceramic cylinders have been analyzed from a viewpoint of considering the anisotropic properties of the ceramic materials, and the results were identical to those introduced by Langevin.¹

The largest sensitivities were associated with shielded-end cylinders made from conventional materials (Ceramic B, PZT-4, and PZT-5A), with the overall maximums for the sensitivities in that category of hydrophones occurring in the very thin-walled cylinders, as expected. The exception to the rule was the lead metaniobate material, which showed the greatest sensitivity for a capped-end cylinder. This material was superior to any of the other materials for equal thickness-to-outside-diameter ratios. However, the lead metaniobate was considered to be unacceptable as an exposed-end cylinder with thin walls because of the tremendous variation in sensitivity in that region.

Axially poled cylinders that were completely pressure-released with respect to the radial coordinate exhibited greater sensitivities than those of similar cylinders that were pressure-released on only one major surface, namely, the inside lateral surface at $r = a$. In an overall sense, the maximum sensitivity was exhibited by the lead metaniobate material. (See Appendix B.)

In addition, it appears that solid bodies can be used as sensitive hydrophone elements without the worry of parameter variations due to hydrostatic pressure loading. In other words, solid-body hydrophones might conceivably be used where stability and uniformity are of paramount importance. Their simplicity in mass production and in multiple-assembly fabrication (no pressure-release problems) are also points that should not be overlooked.

Appendix A

SOME CONSIDERATIONS INVOLVING STATIC STRESSES

The stresses induced in a ceramic cylinder because of ambient hydrostatic pressure (P_h) can be expressed by Eqs. (27), (30), and (31) if P_o is replaced by P_h .

Of the three cases treated in the text of this report, the capped-end cylinder with thin walls ($a/b \rightarrow 1$) induces the maximum axial stress in cylindrical hydrophone elements. A better understanding of the magnitude of the induced axial stress can be obtained by referring to Fig. A-1, which shows a graph of T_3 versus the magnitude of the ambient pressure (P_h) for different values of the thickness-to-outside-diameter ratio (R). This graph was determined by using Eq. (27).

The graph shows that the axial stress is linearly related to the applied load and that the induced stress increases rapidly for equivalent loads as R decreases. For the particular case of an axially polarized ceramic cylinder, the limitation of the induced stress must be interpreted from the viewpoint of the maximum rated static compressive stress (maintained) parallel to the polar axis, which causes failure of the element due to depolarization rather than failure of the element due to fracture. According to the maximum rated value for Ceramic B published by Clevite¹ ($T_{3\max.} = 2000$ psi at 25°C), Fig. A-1 indicates that a cylinder of similar material with $R = 0.2$ should be able to withstand an ambient pressure load of roughly 1.28×10^3 psi. This value corresponds to a depth capability of approximately 2.956×10^3 feet, if it is assumed that the relationship between the ambient pressure and the depth can be expressed by the simple (no temperature dependence or density and gravitational variations) formula

$$P_h (\text{psi}) = (0.433) h \quad (\text{A-1})$$

where h is the submerged depth in feet. For comparison, a similar-type cylinder fabricated with PZT-4 ($T_{1\max.} = 12,000$ psi at 25°C) should be able to withstand quite a bit more than 5000 psi of ambient pressure. Note, however, that even though cylinders can be designed to withstand the expected ambient pressures of their operational environment without

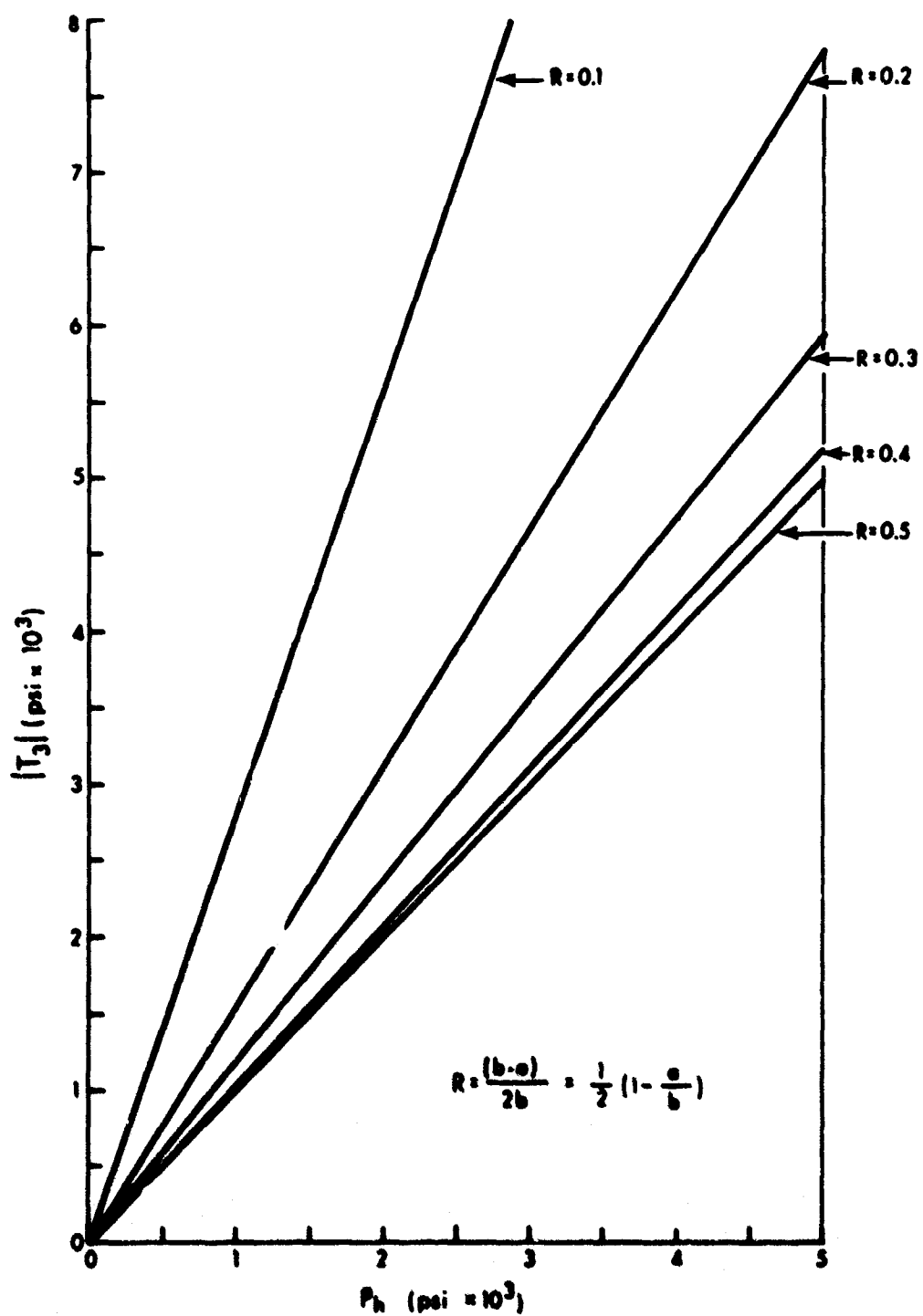


Fig. A-1 - Axial Stress versus Magnitude of the Ambient Pressure for Varying Wall Thicknesses

adverse depolarization effects, the preceding statements are not the final word in good hydrophone design. In other words, if it is stipulated that the sensitivity of the element must be stable with increased pressure, then the possible variation of the material parameters with ambient pressure must be considered. More explicitly, it would seem essential to have knowledge of the variation of the piezoelectric constant (g_{33}) versus internal (or applied) stress in order to predict how the open-circuit sensitivity might vary. For example, if it is assumed that at a specified axial stress level of 1560 psi (corresponding to $P_a = 1000$ psi and $R = 0.2$), the incremental permittivity (ϵ_{33}^T) of PZT-4 increases roughly 4.2 percent and the piezoelectric coefficient (d_{33}) decreases roughly 3.1 percent,¹ then the relative decrease in the piezoelectric constant (g_{33}), which is used directly in determining the open-circuit sensitivity, can be estimated to be 7.0 percent from the relationship

$$g_{33} = \frac{d_{33}}{\epsilon_{33}^T} \quad (A-2)$$

If this stress acted alone, the absolute magnitude of the sensitivity values for the capped-end and exposed-end PZT-4 cylinders would be increased when the cylinders are subjected to an ambient pressure of 1000 psi.

The interpretation given to the stress influence in the above example can not be rigorously applied to the cases treated in the main text of this report because the data from which the result was derived pertained solely to a uniaxial stress. Practical hydrophone elements actually experience a two- or three-dimensional stress distribution when subjected to an ambient pressure. Note, however, that such an interpretation could be legitimately applied to the cases treated in Appendix B.

The main reason for the confusion arises from the fact that radial and tangential stresses, as well as axial stresses, are induced in cylindrical hydrophone elements when they are subjected to ambient pressures. Some of the implications involved when these additional stresses are considered will now be mentioned.

The boundary conditions on the radial stress distribution require it to be equal to zero on the inside lateral surface ($r = a$) and equal to

minus P_h on the outside lateral surface ($r = b$). Therefore, a pressure gradient must exist across the thickness of the cylinder wall, as depicted in Fig. A-2. Since the radial stress is perpendicular to the polarization vector for the cases analyzed in the main text of this report, variations of the transverse piezoelectric coefficient (g_{31}) with respect to the ambient load are of concern here. If the radial stress is temporarily considered to act as an entity by itself, so that at a specified stress level of 1000 psi the incremental permittivity increases roughly 1.7 percent and the transverse piezoelectric coefficient (d_{31}) remains essentially constant, then the relative decrease in the transverse piezoelectric constant (g_{31}) of PZT-4 is calculated to be 1.7 percent. It must be recognized that this decrease in g_{31} occurs only in a small volume of material near the outside peripheral surface and that the material near the inside peripheral surface, since it is unstressed, maintains its original value (measured at zero ambient pressure). In other words, there is a 1.7 percent differential in g_{31} throughout the thickness of the cylinder. For thin-walled cylinders, because of the linear behavior of the stress in that region, this variation can be accounted for by using the value of g_{31} determined at the mean radius, or it can be completely neglected because of the small magnitude of the variation.

The situation is more dramatic when a similar interpretation of the data in Fig. A-2 is applied to radially polarized ferroelectric ceramic cylinders. Under the assumption that Eq. (30), with P_0 replaced by P_h , can be used to represent the radial stress distribution in radially poled cylinders,² it becomes apparent that the material can become inhomogeneous with applied ambient pressure. When the values from the example presented earlier in this appendix are employed, the piezoelectric constant (g_{33}) varies from $g_{33(0)}$ at $r = a$ to 93 percent of $g_{33(0)}$ at $r = b$, where $g_{33(0)}$ is the value of the coefficient at zero ambient pressure. For a thin-walled cylinder, this inhomogeneous phenomenon may be approximately accounted for by considering the piezoelectric constant to possess a mean value between $g_{33(0)}$ and its value when subjected to a 1000-psi parallel stress. This rationale can not be employed for thick-walled cylinders since the radial stress no longer can be considered linear, as indicated in Fig. A-2. Note, however, that for a solid body ($a/b = 0$) the radial stress is uniform and no stress gradient exists. The preceding interpretation given to the effects of the radial stress on the open-circuit sensitivity can not be applied to the cylindrical elements treated in the main text because of

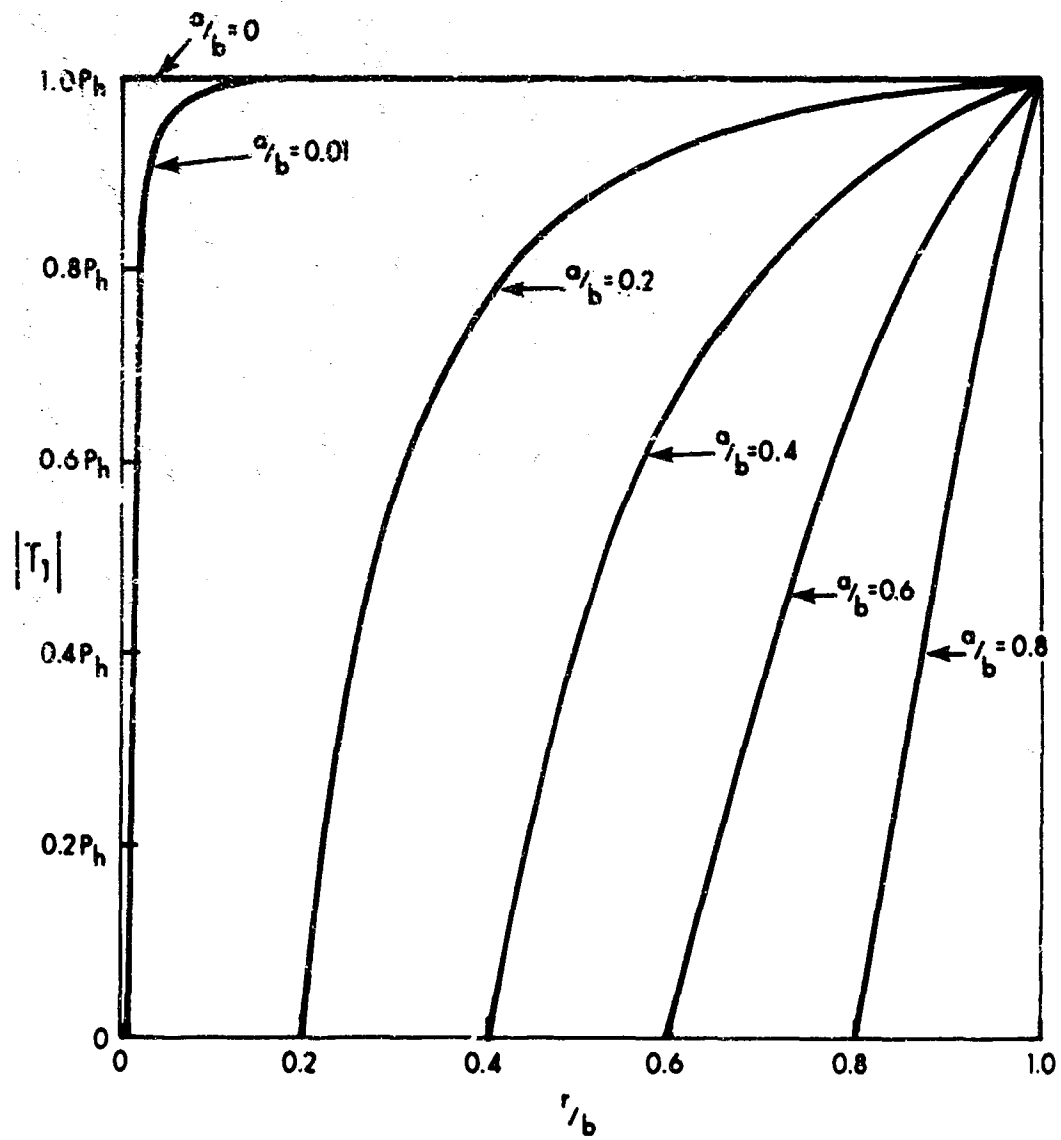


Fig. A-2 - Radial Stress versus Radial Coordinate for Varying Wall Thicknesses

the additional stresses experienced by the cylinder. Primarily, it was included to demonstrate the possibility of introducing material inhomogeneity due to stress gradients induced in the cylinder by ambient pressures.

The final stress distribution that will be considered is that of the tangential (or circumferential) stress. It is given by Eq. (31) with P_o replaced by P_h and will be repeated here for convenience:

$$T_2 = -P_h \frac{\left[1 + \left(\frac{a}{r}\right)^2\right]}{\left[1 - \left(\frac{a}{b}\right)^2\right]} \quad (A-3)$$

For any value of wall thickness, the maximum stress experienced by the cylinder undergoing ambient-pressure variations is at the inside lateral surface ($r = a$) and is tangential in nature. This can be seen by evaluating Eq. (A-3) at $r = a$. The magnitude of the maximum stress is simply twice the magnitude of the axial stress for a capped-end cylinder and, therefore, Fig. A-1 may be used to determine the maximum stress value versus ambient pressure for different values of R if the abscissa is divided by 2. It should now be remembered, however, since the tangential stress is transverse to the polarization direction, that the reconstructed graph must be interpreted in terms of the maximum rated static compressive stress (maintained) perpendicular to the polar axis. For example, if $T_{2\max} = 2000$ psi at 25°C for Ceramic B³, then Fig. A-1 indicates that a cylinder of similar material with $R = 0.2$ would be able to withstand an ambient pressure load of only 0.64×10^3 psi as compared to the 1.28×10^3 psi value, which was previously quoted for the parallel stress. Since this maximum stress is restricted to a small region in the neighborhood of $r = a$, there is more likelihood that the cylinder will experience partial (or local), rather than total, depolarization at high stress levels.

In addition, note that the tangential stress distribution introduces effects that are opposite to those introduced by the radial-stress distribution so far as stress gradients in the ceramic body are concerned. This statement can be appreciated more readily by referring to Fig. A-3, which depicts T_2 versus the ratio of the radial coordinate to the outside radius (r/b) for different values of a/b (ratio of the inside radius to the outside radius). The dashed-line curve represents the magnitude of the maximum stress developed in the cylinder by ambient loads for

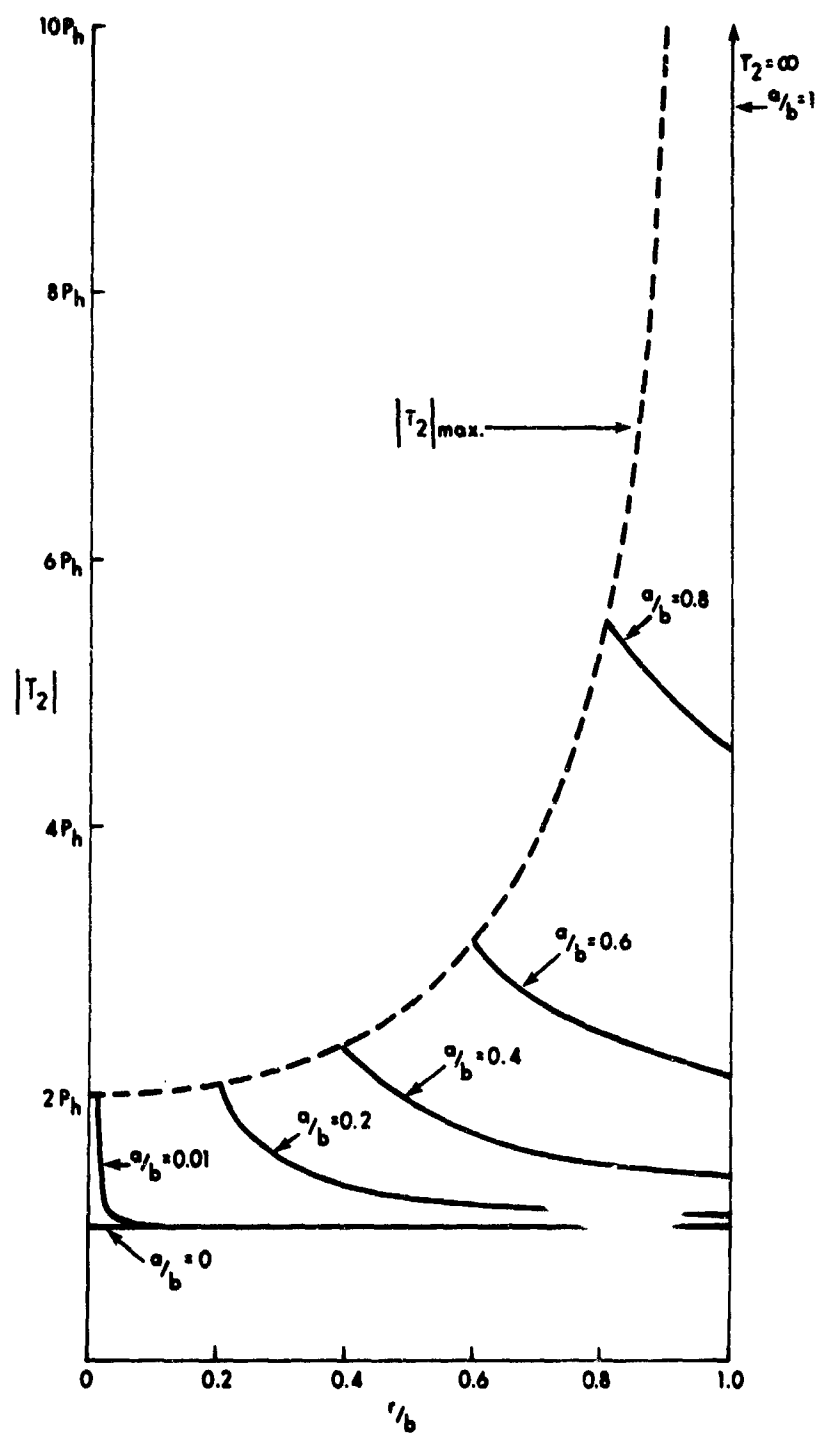


Fig. A-3 - Tangential Stress versus Radial Coordinate for Varying Wall Thicknesses

any wall thickness. Comparison of Figs. A-2 and A-3 reveals that for any set value of a/b the radial stress increases outwardly while the tangential stress increases inwardly, except for the degenerate case of a solid rod ($a/b = 0$) when $T_1 = T_2 = -P_h$.

Now, since the three principal stresses have been analyzed to some extent as separate entities, it would be interesting to speculate on the effects that the combined stresses have on the open-circuit sensitivity of axially poled ceramic cylinders. As an example, let us look at a PZT-4 ceramic cylinder with a wall thickness of $0.4b$ ($R = 0.2$ or $a/b = 0.6$), and let us also assume that the ambient pressure (P_h) is 1000 psi. From Fig. A-3 it is evident that the maximum stress for this particular example is $T_{2\max} = 3.125 P_h = 3125$ psi (compressive), which is the same value that would be determined from Fig. A-1. This stress level is well below the maximum rated static compressive stress (maintained) perpendicular to the polar axis of 8000 psi at 25°C for PZT-4.³ The tangential stress decreases outwardly until at $r = b$ it is equal to 2125 psi, which means that there is a 1000 psi stress differential across the wall thickness of the cylinder. This same result is more easily recognized by referring to Eq. (A-3), which shows that

$$T_2(r=a) - T_2(r=b) = -P_h \quad (A-4)$$

For the above example the radial stress varies from zero psi at $r = 0.6b$ to 1000 psi at $r = b$, and the axial stress has a constant value of 1560 psi throughout the ceramic body. The ratios of the maximum stresses become

$$T_2 : T_3 : T_1 = 3.125 : 1.560 : 1.0 \quad (A-5)$$

At this point in the discussion, the speculation must come to an end. The reason for this is that there are insufficient data on PZT-4 in the published literature to fathom the effects of the above three-dimensional stress distribution on an axially-poled cylinder, especially in view of the fact that T_3 is parallel to the polarization vector.

If the values of the above principal stresses were equal, as in the case of a solid body loaded hydrostatically, then the work of Nishi and Brown⁴² could be used to estimate the changes in ϵ_{33}^T , d_{33} , and d_{31} of PZT-4. For up to a stress level of 10,000 psi in the three principal directions, the net variations in the three parameters were linear

with stress and had the following values at the upper stress level: $\epsilon_{33}^T \rightarrow +4.0$ percent, $d_{33} \rightarrow +5.6$ percent, and $d_{31} \rightarrow +8.1$ percent. The corresponding increases at 1000 psi would, therefore, be +0.4 percent, +0.56 percent, and +0.81 percent, respectively.

It should be mentioned, however, that some data do exist concerning the effects of a three-dimensional stress pattern on a radially polarized cylinder. The ratios of the maximum stresses used by Brown³ in his investigations were

$$T_1 : T_2 : T_3 = 6.67 : 3.33 : 1 \quad , \quad (A-6)$$

where T_3 now represents the radial stress and is in the direction of the polarization vector. At a radial stress (maximum at $r = b$) of roughly 1000 psi, the presented data indicate that the permittivity is essentially constant, with an initial slight upward trend at low stress levels before decreasing substantially with increased stress levels. Unfortunately, the only ceramic parameters that were measured were the free permittivity and the dielectric loss angle. Therefore, no concrete statements can be made concerning the variations of the piezoelectric constant with stress level.

In conclusion, it must be emphasized that more pertinent experimental data are needed before any definitive statements can be made about the overall effect of three-dimensional stresses on the open-circuit sensitivity of either axially-poled or radially-poled cylindrical hydrophone elements.

Appendix B

COMPLETE RADIAL PRESSURE RELEASE OF AXIALLY POLARIZED CERAMIC CYLINDERS

Because of the differences in the piezoelectric constant (g_{33}) associated with the polarization direction in different materials (e.g., lead metaniobate versus Ceramic B), the consideration of the sensitivities that could be obtained by using this coefficient alone should not be overlooked. Therefore, a simple analysis will be performed in order to evaluate the open-circuit sensitivity of a hydrophone element by utilizing the above coefficient as its sole useful parameter. An element of the above type can be envisioned in the form of a cylinder where the boundary conditions on the radial stress distribution are such that $T_1 = 0$ at $r = a$ and $r = b$. This can be achieved by pressure-releasing both the inside and outside lateral surfaces of the cylinder.

Since the basic equations derived in the beginning of this report apply also to the case mentioned above, it will be sufficient to begin with Eq. (20)* and impose the assumed boundary conditions. Therefore, Eq. (20)* requires that

$$0 = a_1 + \frac{a_2}{a^2} \quad \text{and} \quad 0 = a_1 + \frac{a_2}{b^2} \quad (B-1)$$

The above equations are identically satisfied only when $a = b$, which is not a case of any practical importance. For cylinders with finite wall thicknesses, the only solution to Eq. (B-1) is $a_1 = a_2 = 0$. Thus, the radial stress must be zero and, in turn, according to Eq. (21)*, the tangential (or circumferential) stress must be zero also.

The expression for the open-circuit voltage [Eq. (32)*], therefore, becomes simply (remembering that $D_3 = 0$ because of the open-circuit condition)

$$V_{oc} = \int_0^l \mathcal{E}_3 dz = - \int_0^l g_{33} T_3 dz \quad (B-2)$$

*This equation is in the main text of this report.

If the axial stresses proposed for the three cases treated previously in the main text are now considered, it is easily determined that the open-circuit sensitivities can be expressed as follows:

$$\text{Capped-end } M_o = \frac{l g_{33}}{\left[1 - \left(\frac{a}{b}\right)^2\right]}, \quad (\text{B-3})$$

$$\text{Exposed-end } M_o = l g_{33}, \quad (\text{B-4})$$

and

$$\text{Shielded-end } M_o = 0. \quad (\text{B-5})$$

Since the last case is trivial and the second is constant, only the first case has been plotted in Fig. B-1 for the four materials considered herein. A comparison of these sensitivity curves with the ones in the main text indicates that the former sensitivities are larger, even when compared with the shielded-end cases. A more meaningful representation of the differences involved for similarly constructed elements (capped-ends) is shown in Table B-1, where the increase in sensitivity of cylinders employing total radial pressure release over cylinders that are loaded on the outside radial surface is expressed in terms of decibels. The maximum increase was experienced by the PZT-5 material and the minimum increase by the lead metaniobate. However, Fig. B-1 indicates that the overall maximum sensitivity is achieved by using the lead metaniobate material.

Table B-1
INCREASE IN SENSITIVITY PER UNIT LENGTH (M_o/l) OF
EXTERNALLY PRESSURE-RELEASED CYLINDERS
OVER EXTERNALLY LOADED CYLINDERS

Increase in M_o/l (dB)			
Ceramic B	PZT-4	PZT-5	Lead Metaniobate
13.12	16.52	21.83	3.75

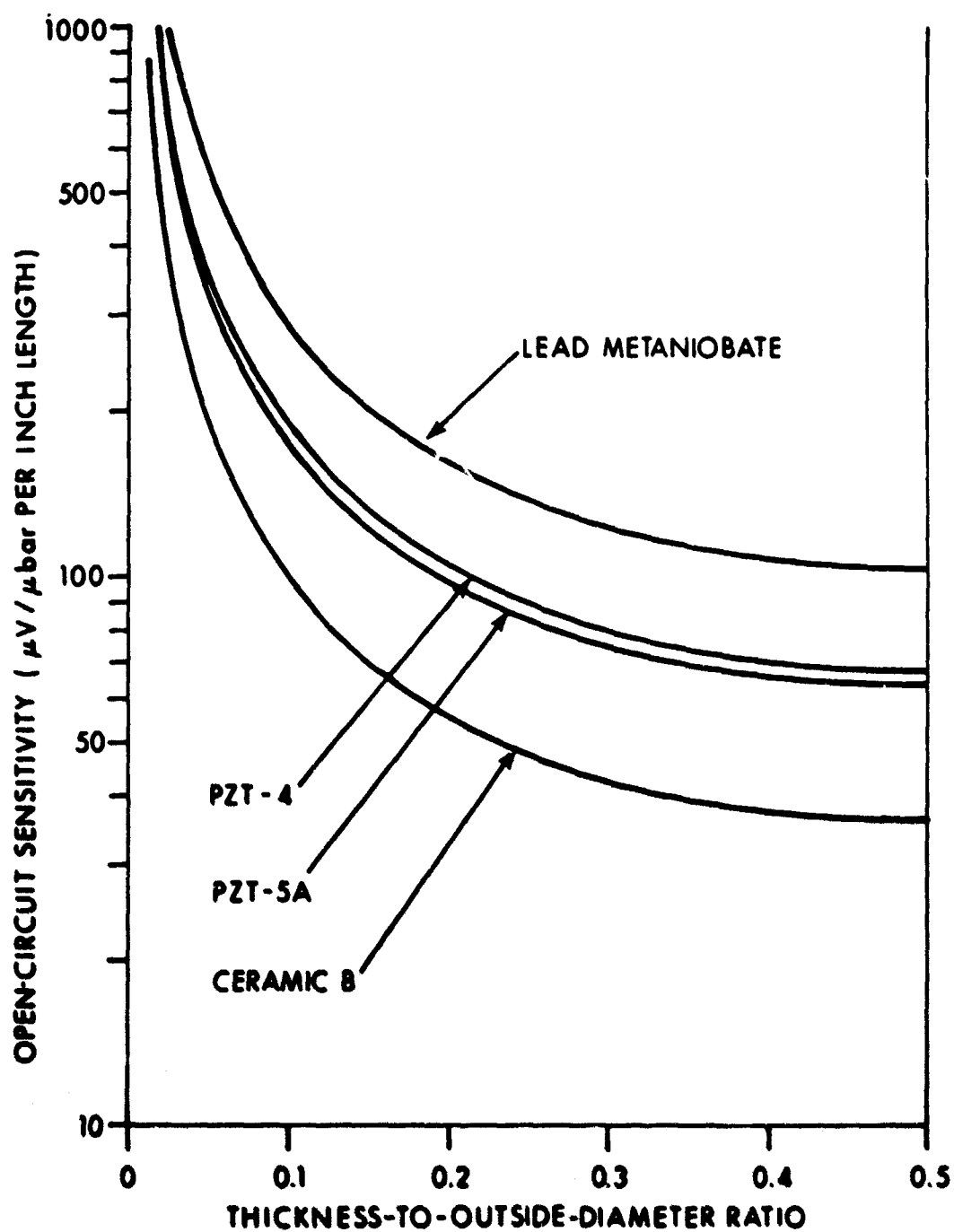


Fig. B-1 - Open-Circuit Sensitivity versus Thickness-to-Outside-Diameter Ratio of Axially Polarized Cylinders (Pressure-Released on the Inside and Outside Surfaces)

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13. ABSTRACT <p>The open-circuit sensitivities of axially polarized ferroelectric ceramic cylinders are derived by treating the ceramic as an anisotropic material. Although the results for cylinders that are pressure-released at the inside lateral surface were identical to those given by Langevin, the analysis showed that the anisotropic properties of currently used ceramics did not influence the sensitivities of these types of cylindrical hydrophone elements.</p> <p>Axially poled cylinders that were completely pressure-released with respect to the radial coordinates were found to exhibit sensitivities larger than those of similar cylinders that were pressure-released on only one major surface, the inside lateral surface. Included in this report are graphs of the magnitude of the stresses induced in the cylinders by the ambient hydrostatic pressures.</p>		

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